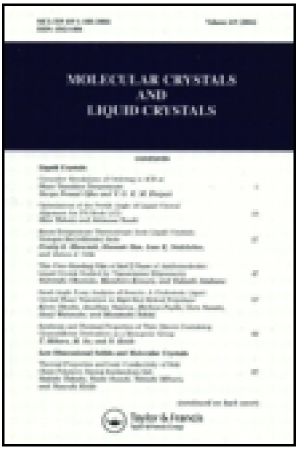

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Generalized Freedericksz Transition in Polymer Nematics

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GENERALIZED FREEDERICKSZ TRANSITION IN POLYMER NEMATICS

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Abstract. Recent theoretical studies on the relative occurrence of field induced static periodic domains **(PD)** and homogeneous deformation **(HD)** are reviewed for different sample and field configurations. Apart from energetics the related language of torques can be employed to obtain a qualitative explanation of the destabilizing positive feedback mechanism **(PFM)** which causes PD to develop in materials having high elastic anisotropy. PD may be suppressed by a proper choice of field direction, initial director orientation or director anchoring strengths at the boundaries; simultaneous application of an orthogonal stabilizing field may also quench PD. In a simple twisted nematic, PD may occur as one of two independent modes associated with orthogonal directions of periodicity. Solutions reminiscent of PD result for biaxial nematics though in this case PD may have a more complex form. When the sample is cylindrical and the field radial PD may result, depending upon the initial director orientation, either as a static Taylor instability or as a distortion whose azimuthal variation is governed by a dimensionless wavevector taking integral values. Though, in the limit of weak dielectric anisotropy and zero electrical conductivity, analysis for electric field induced PD is similar to that of the magnetic case the importance of flexoelectricity and non-local interaction of the field cannot be ruled out.

INTRODUCTION

The anisotropic elastic properties of uniaxial nematics are adequately described by the Oseen-Frank continuum theory of curvature elasticity[1-7] in which the bulk elastic free energy density W_V is written as a quadratic in the spatial gradients of the unit director vector field $\mathbf{n}$ representing the average direction of molecular orientation at any given point of the material. W_V also depends on the elastic constants K_1, K_2, K_3 corresponding, respectivley to splay, twist and bend deformations of **n**. Suitable treatment of sample walls can help impart different director orientations and also influence the surface director anchoring energy density W_σ; one of the simplest and most widely used forms of W_σ was proposed by Rapini and Papoular.[8-10] Perfectly aligned nematic samples (with $\mathbf{n}=\mathbf{n}_o$ = constant), corresponding to minimum free energy, can be prepared by suitable treatment of the sample boundaries. In such a sample the director field can be distorted by applying a magnetic field **H** which produces a disrupting torque via the diamagnetic susceptibility anisotropy χ_a of the nematic; elastic restoring torques enable **n** to reach an equilibrium configuration. When **H** is normal to the plates of a nematic sample with $\mathbf{n}_o$ parallel to the plates (splay geometry), $\chi_a > 0$ and K_1 not much larger than K_2, **H** creates a destabilizing torque by coupling with fluctuations in $\mathbf{n}_o$; this leads to the formation of a splay distortion which is uniform in the sample plane (homogeneous distortion, HD) when $|H| \gtrsim H_H$, the splay Freedericksz threshold. K_1 can be evaluated by determining H_H; K_2 and K_3 can be similarly evaluated from other geometries. The oblique field configuration can also be used[11] by applying **H** in a plane normal to $\mathbf{n}_o$; in this case HD is a mixture of more than one kind of fundamental deformation.

When Lonberg and Meyer[12] studied the polymer nematic (PN) PBG in the splay geometry they observed, above a well defined field threshold, a distortion having periodicity in the sample plane along a direction roughly normal to $\mathbf{n}_o$, the ground state. Using the Oseen-Frank continuum theory of uniaxial nematics they showed[12] that the periodic distortion (PD) occurs in PNs like PBG due to high elastic anisotropy of these materials[13,14,15] resulting from rather unusual molecular dimensions. In such systems, as PD has lower threshold than the splay homogeneous deformation (HD), the conventional method of measuring the splay constant K_1 cannot be used. During the last two years some theoretical attempts have been made[16-21] to study the occurrence of PD in different configurations and also suggest ways in which PD can be quenched. A brief review of the results is presented below with emphasis on qualitative understanding.

PLANAR ORIENTATION

Consider a rigidly anchored nematic in the splay geometry with $\mathbf{n}_o=(1,0,0)$ between plates $z=\pm h$ acted upon by a magnetic field $\mathbf{H}=(0,0,H_z)$; let the diamagnetic anisotropy $\chi_a>0$. Under perturbation let $\mathbf{n}_o \rightarrow \mathbf{n}=(1-\theta^2/2-\phi^2/2,\phi,\theta)$ where θ,ϕ are functions of x,y,z. The total free energy density W_T is given by

$$W_T = W_H + W_S + W_C;\ 2W_H = K_1\theta_{,z}^2 - \chi_a H_z^2\theta^2;$$
$$W_C = K_1\theta_{,z}\phi_{,y} - K_2\theta_{,y}\phi_{,z} \ ;$$
$$2W_S = K_2(\phi_{,z}^2 + \theta_{,y}^2) + K_1\phi_{,y}^2 + K_3(\phi_{,x}^2 + \theta_{,x}^2) \qquad (1)$$

where $\phi_{,z}=\partial\phi/\partial z$, etc. K_2, K_3 are the twist and bend constants, respectively. The total free energy F_T is the integral of W_T over the sample volume. If we put $\theta=\theta(z)$, $\phi=\phi(z)$ and extremize W_H with respect to θ,ϕ with the(rigid anchoring) boundary conditions

$$\theta(\pm h) = 0 = \phi(\pm h) \qquad (2)$$

we get the HD (splay Freedericksz) threshold

$$H_H = (q_z/h)\,(K_1/\chi_a)^{1/2} \qquad (3)$$

with $q_z=\pi/2$; it is also found that $\phi\approx 0$ for $H_z \gtrsim H_H$.

By asserting that the nematic undergoes PD in preference to HD, we demand that (i) the distortion above threshold must have θ and ϕ depending on both y and z; (ii) such a distortion must have lower free energy than HD. It is seen that were θ, ϕ to depend upon x, z a purely positive contribution would be added to make W_T always higher than W_H. When θ, ϕ are functions of y, z, though a positive W_S is added to W_H, cross terms W_C are also present; these can, in principle, bring down F_T to a level less than the total free energy F_H for HD via a negative contribution of sufficient magnitude. Figuratively, W_C serves as a conduit to carry away excess free energy associated with a pure splay HD into the twist of PD. Equivalently, if the director can bring down the total free energy by escaping out of the xz plane via a twist, there must exist a destabilizing positive feedback mechanism (PFM)[22] which makes θ unstable against ϕ. To see this, W_T is extremized with respect to θ, ϕ to yield the torque equations

$$-\Gamma_\theta = K_1\theta,_{zz} + K_2\theta,_{yy} + \chi_a H_z^2\theta + k_{12}\phi,_{yz} + K_3\theta,_{xx} = 0;$$

$$\Gamma_\phi = K_2\phi,_{zz} + K_1\phi,_{yy} + k_{12}\theta,_{yz} + K_3\phi,_{xx} = 0 , \qquad (4)$$

where $k_{12} = K_1 - K_2$. The following points may be noted: (i) Assuming solutions periodic in x of the form $\cos(q_x x)$ and $\sin(q_x x)$ for θ, ϕ respectively, leads to purely stabilizing torques. Hence, x dependence is ignored; then (4) go over to the corresponding equations of ref. 12. (ii) Eq.(4) supports two independent modes with respect to z variation -

Mode Y_1 : θ_e even, ϕ_o odd; **Mode Y_2**: θ_o odd, ϕ_e even.

(iii) For either mode, when θ, ϕ are periodic along y they are also exactly out of phase. (iv) When $K_1 \gg K_2$, Mode Y_1 is less energetic than Mode Y_2. Solutions are sought for Mode

Y_1 in the form,

$$(\theta,\Gamma_\theta)=(\theta_A,\ \Gamma_{\theta A})C_{qz}C_{qy};\ (\phi,\Gamma_\phi)=(\phi_A,\Gamma_{\phi A})S_{qz}S_{qy}\ ;$$
$$S_{qz}=\sin q_z z;\ S_{qy}=\sin q_y y;\ C_{qz}=\cos q_z z;$$
$$C_{qy}=\cos q_y y;\ \theta_A,\ q_y,\ q_z>0 \tag{5}$$

without loss of generality. One finds from (4) that θ gives rise to a ϕ via the torque $\Gamma^{(1)}_{\phi A}\sim k_{12}\theta_A q_y q_z$ such that $\phi_A\sim k_{12}\theta_A q_y q_z/(K_1 q_y^2+K_2 q_z^2)>0$ if $k_{12}>0$. The ϕ, in turn, causes the torque $\Gamma^{(1)}_{\theta A}\sim -k_{12}\phi_A q_y q_z<0$ which, having the same sign as the destabilizing magnetic torque $\Gamma^{(2)}_{\theta A}\sim -\chi_a H_z^2\ \theta_A$, further enhances the original θ ; the PFM is completed. The purely restoring elastic torques $\Gamma^{(3)}_{\theta A}\sim(K_1q_z^2+K_2q_y^2)\theta_A$ and $\Gamma^{(2)}_{\phi A}\sim -(K_2q_z^2+K_1q_y^2)\ \phi_A$ ensure a threshold for PD. The following points become clear: (i) For PFM to work, $k_{12}>0$. (ii) The parity of perturbations is such that if $\theta_A>0$, $\phi_A>0$. (iii) The magnitude of $\Gamma^{(1)}_{\theta A}\sim\phi_A$; larger the ϕ_A, greater the destabilization. Using the above ansatz and integrating W_C over $z(-h,+h)$ and $y(0,\ \pi/q_y$; half the wavelength or one domain width) one finds the energy F_C associated with the cross terms:

$$F_C\sim -k_{12}\zeta+(K_1+K_2)\sin\zeta;\ \zeta=2q_zh\ . \tag{6}$$

When θ is rigidly anchored, $\zeta\sim\pi$; then $F_C<0$ if $k_{12}>0$. Thus the cross terms are found to make a negative contribution towards F_T when θ is rigidly anchored. Exact numerical calculation[12] shows that when

$$K_2/K_1<\alpha;\ \ \alpha\approx 0.3 \tag{7}$$

the PD threshold $H_{zP}<H_H$ and the domain wave vector at PD threshold $q_{yc}\neq 0$. When $K_2/K_1\to\alpha$, $H_{zP}\to H_H$ and $q_{yc}\to 0$ so that for $K_2/K_1>\alpha$, HD is more favourable than PD. Perturbation analysis around the critical point[17,18,20] yields the more exact expression

$$\alpha = [\beta^2+\beta]^{1/2} - \beta; \beta = (\pi^2/8)-1 . \tag{8}$$

It has been recognized[16-18] that PD can occur in a system with $K_2 > K_1$ (for instance, a nematic close to a smectic transition)[3,5,7] when $\mathbf{H}=(0,H_y,0)$. In this case, $H_z=0$ in (4) and the term $\chi_a H_y^2 \phi$ is added to Γ_ϕ. It turns out that Mode Y_2 has lower threshold than Mode Y_1. Due to a symmetry transformation linking the two cases of H_y and H_z fields, the Mode Y_2 threshold H_{yP} for $K_2/K_1 = \alpha_1$ is identical to the Mode Y_1 threshold H_{zP} for $K_1/K_2 = \alpha_1$; the threshold wavevector in both cases is the same.

Effects of different parameters on the relative occurrence of PD and HD will be summarized below for planar orientation.

Anchoring Energy

The Rapini-Papoular expression for the surface energy density[8-10] for a nematic in the splay geometry is

$$W_\sigma = [B_\theta\theta^2 + B_\phi\phi^2](z=+h) + [\bar{B}_\theta\theta^2 + \bar{B}_\phi\phi^2](z=-h) \tag{9}$$

where B_θ, $\bar{B}_\theta$ are splay anchoring strengths and B_ϕ, $\bar{B}_\phi$ twist anchoring strengths, respectively. It is convenient to consider symmetric anchoring such that

$$B_\theta = \bar{B}_\theta ; \quad B_\phi = \bar{B}_\phi . \tag{10}$$

Using (9) and (10) and (1) it is found[17-20] that as B_θ, B_ϕ determine boundary conditions their relative magnitudes can greatly affect the relative field magnitude $R_H = H_{zP}/H_H$ as well as q_{yc}. Results go over to the rigid anchoring limit (2) only when B_θ, B_ϕ are large enough. H_H is given by (3) with $q_z = q_z(B_\theta)$; $q_z \to 0$ as $B_\theta \to 0$. Similarly, when B_θ, B_ϕ are decreased, H_{zP} and q_{yc} also diminish. However, the rates of decrease of H_{zP} and H_H vary so that anchoring energy can be used as a tool for controlling the domains of existence of PD and HD. In particular, α (Eq.7) becomes a function of B_θ, B_ϕ.

When B_θ is held constant at a high value for a given material and B_ϕ is diminished, H_H remains unchanged while H_{zP} decreases. A slackening of anchoring on ϕ aids the formation of PD albeit with a larger domain size. In the limit of $B_\phi \to 0$, α increases from ≈ 0.3 to 0.5.

When, for a given material, B_ϕ is large (ϕ firmly anchored) and B_θ is diminished, both H_{zP} and H_H decrease. However the rate of decrease of H_{zP} is slower than that of H_H as PD is associated with an extra degree of freedom which is firmly anchored. When B_θ becomes sufficiently small, $H_{zP} \to H_H$ and $q_{yc} \to 0$; for sufficiently weak anchoring of θ, HD should become more favourable than PD. This case corresponds to $\zeta \to 0$ in (6); in this limit, $F_c \to 0$ showing that the cross terms make a negligible negative contribution. Needless to say, α decreases from (8) when B_θ is diminished.

Oblique Field

$\mathbf{H}=(0, H_N S_\xi, H_N C_\xi)$; $S_\xi = \sin\xi$; $C_\xi = \cos\xi$; the field is applied obliquely in the yz plane making angle ξ with z axis. W_T is given by (1) with $-\chi_a H_N^2(S_\xi\phi + C_\xi\theta)^2$ replacing $-\chi_a H_z^2\theta^2$ in $2W_H$. In the torque eqns. (4) we replace $\chi_a H_z^2\theta$ by $\chi_a H_N^2(S_\xi\theta + C_\xi\theta)C_\xi$ and add $\chi_a H_N^2(S_\xi\theta + C_\xi\theta)S_\xi$ to Γ_ϕ. Clearly, the limit $\xi \to 0$ leads to the case of the H_z field. Putting $\theta = \theta(z)$, $\phi = \phi(z)$, extremizing W_H with respect to θ, ϕ and using (2) the HD threshold for rigid anchoring is found to be $H_{NH}(\xi) = (\pi/2h)[K_1K_2/\chi_a(K_1S_\xi^2 + K_2C_\xi^2)]^{1/2}$; for $H_N \gtrsim H_{NH}$, HD is associated with even θ_e and ϕ_e.

The effect of oblique field on PD is quite different.[16,18,19]

Though for $\xi \to 0$ the PD threshold $H_{NP}(\xi) \to H_{zP}$, for $\xi \neq 0$, it is not possible to have a pure modal deformation. The distortion is a superposition of Modes Y_1 and Y_2 with $\theta = \theta_e(z)C_{qy} + \theta_o(z)S_{qy}$; $\phi = \phi_o(z)S_{qy} + \phi_e(z)C_{qy}$. The effect of mode mixing is to increase the elastic free energy of PD wrt that of HD by adding the higher energy Mode Y_2 to PD; obviously, when ξ is increased from zero, the proportion of Mode Y_2 also increases so that $R_H(\xi) = H_{NP}(\xi)/H_{NH}(\xi)$ increases and the domain wavevector $q_{yc}(\xi)$ decreases. When $\xi \to \xi_c(K_1, K_2)$, $R_H(\xi) \to 1$ so that HD should be observable for $\xi > \xi_c$. For PBG[13,15] with $K_1:K_2:K_3::11.4:1:13$, $\xi_c \approx 0.35$ radian. Needless to say ξ_c is an increasing function of K_1/K_2.

OBLIQUE ORIENTATION

Let the nematic be obliquely oriented in the sample such that $\mathbf{n}_o = (C_\varphi, 0, S_\varphi)$; φ = constant; $0 \leq \varphi \leq \pi/2$. In addition, let $\mathbf{n}$ be rigidly anchored at the sample boundaries $z = \pm h$. The field $\mathbf{H} = H_N(-S_\varphi, 0, C_\varphi)$ acts in the xz plane and is normal to $\mathbf{n}_o$. Under perturbation, $\mathbf{n}_o \to \mathbf{n} = (C_\varphi - S_\varphi\theta, \phi, S_\varphi + C_\varphi\theta)$. Expressions for W_T and torques can be set up as in (1) and (4); these are found to contain additional terms depending on K_3 apart from coefficients which are functions of φ. If $\theta = \theta(z)$, $\phi = \phi(z)$, then at HD threshold $H_{NH}(\varphi) = (\pi/2h)[(K_1C_\varphi^2 + K_3S_\varphi^2)/\chi_a]^{1/2}$, $\phi \approx 0$. When θ, ϕ are functions of y,z, PD is found to have the same symmetry as Mode Y_1. Using the ansatz (5) one can locate a very similar PFM which should cause Mode Y_1 PD. However, there are some changes; $\phi_A \sim k_{12}C_\varphi q_y q_z \theta_A/[K_1q_y^2 + (K_2C_\varphi^2 + K_3S_\varphi^2)q_z^2]$ showing that ϕ_A decreases when K_3 or φ is augmented. The destabilizing torque $\Gamma_{\theta A}^{(1)} \sim -k_{12}C_\varphi q_y q_z \phi_A$ also decreases when K_3 or φ is enhanced. The free energy $F_c(\varphi)$ associated with the cross terms $\sim C_\varphi F_c(0)$ where $F_c(0)$ is given by (6). As φ is increased, $F_c(\varphi)$ makes less negative contribution. The effect of all this increases the PD threshold $H_{NP}(\varphi)$ at a rate faster than $H_{NH}(\varphi)$ when φ or K_3 is increased; this also decreases the PD threshold wavevector $q_{yc}(\varphi)$. Exact numerical calculation shows[16,19] that when $\varphi \to \varphi_c(K_1,K_2,K_3)$, $H_{NP}(\varphi) \to H_{NH}(\varphi)$ and $q_{yc}(\varphi) \to 0$; HD should be possible for $\varphi > \varphi_c$. φ_c increases when K_1/K_2 is increased or when K_3/K_2 is decreased. For PBG, $\varphi_c \approx 0.9$ radian.

CROSSED ELECTRIC AND MAGNETIC FIELDS

It has been suggested[18] that the effect of an electric field $\mathbf{E}=(0,0,E_z)$ on PD is analogous to that of the H_z field in the splay geometry for a non-conducting nematic; we replace $-\chi_a H_z^2\theta$ in (1) by $-\varepsilon_a E_z^2\theta/4\pi$ where $\varepsilon_a(>0)$ is the dielectric susceptibility anisotropy. Such analogy is restricted to small ε_a as will be discussed later. It also seems possible[18] that simultaneous application of $\mathbf{E}$ and $\mathbf{H}=(H_x,0,0)$ may quench PD for $H_x >$ a critical value $H_{xc}(K_1,K_2)$ if $\chi_a>0$. In this case one adds the stabilizing torques $\chi_a H_x^2\theta$ and $-\chi_a H_x^2\phi$ to Γ_θ and Γ_ϕ, respectively. The clamping of ϕ by H_x reduces the amplitude of ϕ to $\phi_A \sim k_{12}\theta_A q_y q_z/(K_1 q_y^2+K_2 q_z^2+\chi_a H_x^2)$ and subdues PFM; the presence of $\mathbf{H}$ also adds a positive contribution $\sim\chi_a H_x^2(\theta^2+\phi^2)/2$ to the PD free energy. Consequently, the PD threshold E_{zP} increases faster than the HD threshold $E_{zH}=[(4\pi K_1/\varepsilon_a h^2)\{(\pi^2/4)+(\chi_a h^2 H_x^2/K_1)\}]^{1/2}$ when H_x is increased so that when $H_x \to H_{xc}(K_1,K_2)$, $E_{zP}(H_x)\to E_{zH}(H_x)$ and $q_{yc}(H_x)\to 0$; for $H_x > H_{xc}$, HD should be observable. Clearly, H_{xc} is an increasing function of K_1/K_2. One can also expect a decrease of E_{zP} wrt E_{zH} when a field $(0,H_y,0)$ is applied with $H_y <$ the twist Freedericksz threshold.

It is difficult to think of electric field effects in nematics without considering flexoelectricity.[6,23,24] One of the main effects of flexoelecticity is to introduce large elastic anisotropy for certain configurations.[25] As elastic anisotropy is an important factor in determining the relative occurrence of PD and HD, it seems necessary to include flexoelectricity in a theoretical study. Another important factor is the non-local effect of **E**.[6,26] The director perturbations cause a fluctuation **E'** in **E**. The effect of **E'** must be considered fully by including it in the free energy density (1). These investigations will be reported separately.

SIMPLE TWISTED NEMATIC

The ground state is $\mathbf{n}_o = (\cos Tz, \sin Tz, 0)$ between plates $z = \pm h$ such that $T = \gamma/h$; $0 \leq \gamma < \pi/4$. Under the action of $\mathbf{H} = (0,0,H_z)$, HD threshold[27] is given by $H_{zH} = [\{K_1\pi^2 + 4\gamma^2(K_3 - 2K_2)\}/4h^2 \chi_a]^{1/2}$. With perturbations θ, ϕ, $\mathbf{n}_o \rightarrow \mathbf{n} = [\cos(Tz+\phi)\cos\theta, \sin(Tz+\phi)\cos\theta, \sin\theta]$; torque equations can be set up and solved with suitable boundary conditions. It is found that PD can occur (as one of two modes) with lower threshold than HD over certain ranges of parameters.[21] As $\mathbf{n}_o = (1,0,0)$ at $z = 0$, it is possible to think of two independent PD modes: (i) Mode Y_1 with θ, ϕ having the same structure as in the planar untwisted case and periodicity along y; (ii) Mode X_1 with periodicity along x and having both θ, ϕ even.

For rigid anchoring, γ the ground state twist and K_3 are additional parameters. The existence of Modes X_1 and Y_1 depends critically on the magnitudes of γ, K_1, K_2, K_3. Mode Y_1 is found to prevail when K_1/K_2 is high and γ, K_3/K_2 are small. Mode X_1 is found to be favourable when γ is large and K_1 small compared to K_3. Thus, in PBG, Mode Y_1 should exist over almost the entire γ range while in TMV,[15] Mode X_1 should prevail over a broader γ range. As Mode X_1 does not conform to the symmetry of $\mathbf{n}_o$, the Mode X_1 domain size is found to be generally larger than that of Mode Y_1. Variation of γ for a suitable material may result in crossover from Mode Y_1 to Mode X_1 or vice versa. Such a crossover, accompanied by changes in the direction of periodicity and domain size, may be experimentally observable.

When anchoring is weak, the thresholds and domain sizes of both PD modes depend on the additional parameters B_θ and B_ϕ.[28] In particular, a decrease in B_ϕ aids (deters) the formation of Mode Y_1 (Mode X_1); a diminution of B_θ is helpful (detrimental) to the formation of Mode X_1 (Mode Y_1). However, when B_θ is very small and γ not large it seems possible to suppress both PD Modes; HD should be observable. Results for the twisted nematic case can be qualitatively explained as in the case of the planr untwisted sample.

BIAXIAL NEMATIC

After the first discovery[29] of the biaxial nematic (**BN**) phase over a narrow thermodynamic range in a lyotropic system, Saupe[30] generalized the Oseen-Frank theory to describe the elastic free energy density of an orthorhombic BN as a quadratic in the gradients of three orthonormal director fields **a**, **b**, **c** and depending upon twelve elastic coefficients. The subsequent discoveries of a thermotropic BN[31] and of the probable optical biaxiality of certain polymer systems[32,33] make it necessary to investigate whether solutions corresponding to PD are possible in BN.

Torque equations written using Saupe's theory in the small deformation limit do not rule out the possibility of a uniformly aligned BN with one director field normal to the plates $z=\pm h$ and field along z, undergoing a grid-like instability with periodicity along x and y. Due to the large number of free parameters, an analysis of this case becomes difficult. When a simplifying assumption is made that periodicity is along x or y, the torque equations take a form isomorphic to (4). It can then be shown that PD should be more favourable than HD over certain ranges of the elastic constants[28] for rigid anchoring of director fields.

CYLINDRICAL GEOMETRY

It is known[34,35] that a uniaxial nematic aligned suitably between coaxial cylinders of radii R_1, R_2 undergoes HD when the magnitude of the impressed radial field $\mathbf{H}_R=(A/r,0,0)$ is sufficiently high (A=constant). In particular, when the initial alignment is azimuthal ($n_r=n_z=0$, $n_\Psi=1$; r,Ψ,z are cylindrical polar coordinates; z is the common axis of the cylinders) the director may undergo HD spontaneously, even in the absence of field, when the radial ratio $R_{21}=R_2/R_1$ exceeds a geometrical threshold (**GT**). The existence of GT can be traced to the global deformation energy $\sim K_3 \ln(R_{21})$ which can be diminished by a distortion which brings in the smaller splay constant K_1. When the initial alignment is axial ($n_r=n_\Psi=0$, $n_z=1$), GT does not exist as the director field is not globally distorted.

It is straightforward to guess[28] that when $K_1 \gg K_2$ and the initial alignment azimuthal, $\mathbf{H}_R$ should produce PD with modulation along z similar in form to the Taylor instability[36] in isotropic liquids. For certain ranges of material parameters, PD even possesses GT lower than that of HD. When the initial alignment is axial, $\mathbf{H}_R$ can induce PD with modulation along Ψ with a dimensionless wavevector $q_{\Psi c}$ which is a measure of the number of domains and hence integral. When material parameters and R_{21} are varied, $q_{\Psi c}$ exhibits discrete jumps reminiscent of the Barkhausen effect in ferromagnetic materials.[37]

Despite theoretical complications the effect of a radial electric field may be experimentally convenient.

In conclusion, the continuum theory indicates different methods for encouraging and discouraging the formation of PD in PNs of high elastic anisotropy. PD may also appear in different forms depending upon sample and field configurations. It may be interesting to check these results experimentally. In the static limit the well known torque formalism[22] indicates the PFM which can make PD more favourable than HD over certain ranges of parameters. A complete discussion, however, would involve viscous effects.

REFERENCES

1. C.W.Oseen, Trans. Faraday Soc. **29**, 883 (1983).
2. F.C.Frank, Disc. Faraday Soc. **25**, 19 (1958).
3. P.G. de Gennes, The Physics of Liquid Crystals (Clarendon Press, Oxford, 1974).
4. J.L.Ericksen, in Advances in Liquid Crystals, edited by G.H.Brown (Academic Press, 1976), Vol. 2, pp. 233-298.
5. S. Chandrasekhar, Liquid Crystals (Cambridge University Press, Cambridge, 1977).
6. H.J.Deuling, Solid State Physics, Suppl. **14**, 77 (1978).
7. L.M.Blinov, Electro-optical and Magneto-optical Properties of Liquid Crystals (John Wiley, 1983).
8. A. Rapini and M.Papoular, J. Physique Colloq. **30**, C4-54 (1969).
9. E.Guyon and W.Urbach, in Nonemissive Electrooptical Displays, edited by A.R.Kmetz and F.K. von Willisen (Plenum Press, 1976), p. 121.
10. J.Cognard, Mol. Cryst. Liquid Cryst. Suppl. **1** (1982).
11. H.J.Deuling, M.Gabay, E.Guyon and P.Pieranski, J. Physique, **36**, 689 (1975).
12. F.Lonberg and R.B.Meyer, Phys. Rev.Lett., **55**, 718 (1985).
13. V.G.Taratuta, A.J.Hurd and R.B.Meyer, Phys. Rev. Lett., **55**, 246 (1985).
14. R.B.Meyer, in Polymer Liquid Crystals, edited by A.Ciferri, W.R.Krigbaum and R.B.Meyer (Academic Press, 1982), pp. 133-163.
15. R.B.Meyer, F.Lonberg, V.Taratuta, S.Fraden, S.D.Lee and A.J.Hurd, Faraday Disc. Chem. Soc., **79**, 125 (1985).

16. U.D.Kini, J. Physique, **47**, 693 (1986).
17. C.Oldano, Phys. Rev. Lett., **56**, 1098 (1986).
18. W.Zimmerman and L.Kramer, Phys.Rev.Lett., **56**, 2655 (1986).
19. U.D.Kini, J. Physique, **47**, 1829 (1986).
20. E.Miraldi, C.Oldano and A.Strigazzi, Phys. Rev., **A 34**, 4348 (1986).
21. U.D.Kini, J. Physique **48**, 1187 (1987).
22. E.Dubois-Violette, E.Guyon, I.Janossy, P.Pieranski and P.Manneville, Solid State Physics Suppl. **14**, 147 (1978).
23. R.B.Meyer, Phys. Rev. Lett., **22**, 918 (1969).
24. G.Durand, Mol. Cryst. Liquid Cryst., **113**, 237 (1984).
25. G.Barbero and M.Meuti, J. Physique, **47**, 341 (1986).
26. I.Dozov, G.Barbero, J.F.Palierne and G.Durand, Europhys. Lett., **1**, 563 (1986).
27. F.M.Leslie, Mol. Cryst. Liquid Cryst., **12**, 57 (1970).
28. U.D.Kini, J. Physique (submitted).
29. L.J.Yu and A. Saupe, Phys. Rev. Lett., **45**, 1000 (1980).
30. A.Saupe, J. Chem. Phys., **75**, 5118 (1981).
31. J.Malthete, L.Liebert, A.M.Levelut and Y.Galern, C. R. Acad. Sc. Paris, **303** Series II, 1073 (1986).
32. C.Viney, G.R.Mitchell and A.H.Windle, Mol. Cryst. Liquid Cryst., **129**, 75 (1985).
33. A.H.Windle, C.Viney, R.Golombok, A.M.Donald and G.R. Mitchell, Faraday Disc. Chem. Soc., **79**, 55 (1985).
34. F.M.Leslie, J. Phys., **D 3**, 889 (1970).
35. R.J.Atkin and P.J.Barratt, Q. J. Mech. Appl. Math., **26**, 109 (1973).
36. A.R.Paterson, A First Course in Fluid Dynamics (Cambridge University Press, 1983).
37. R.M.Bozorth, Ferromagnetism (D. Van Nostrand, 1951).